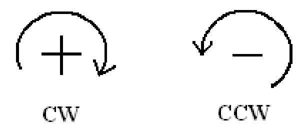
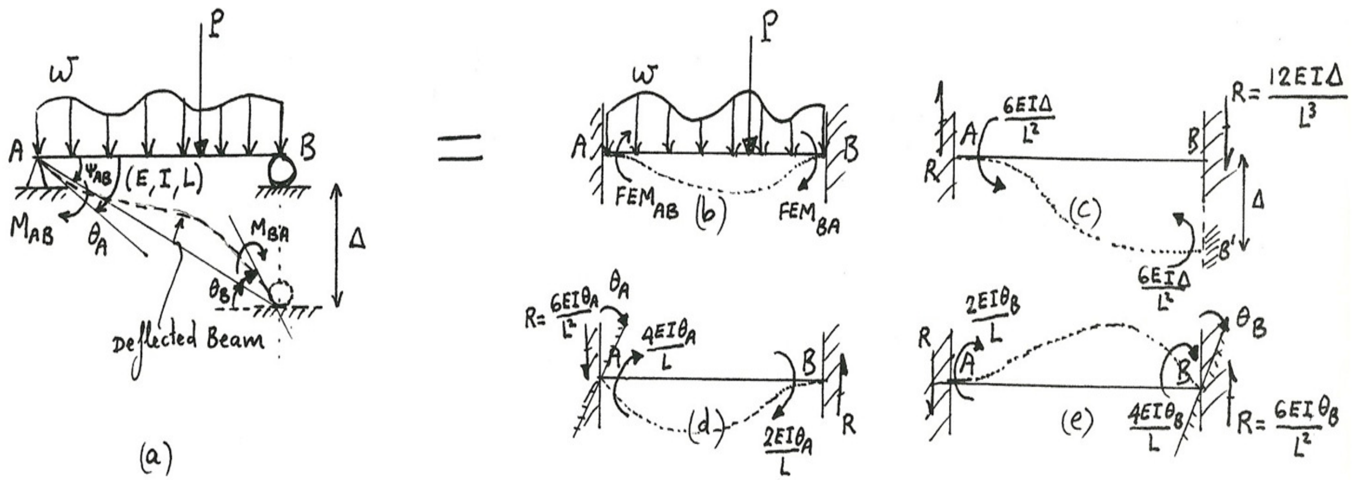
SLOPE DEFLECTION EQUATION





The end-moments of Figure (a) = $\sum$(end Moments of Figures b + c + d + e)

Thus:

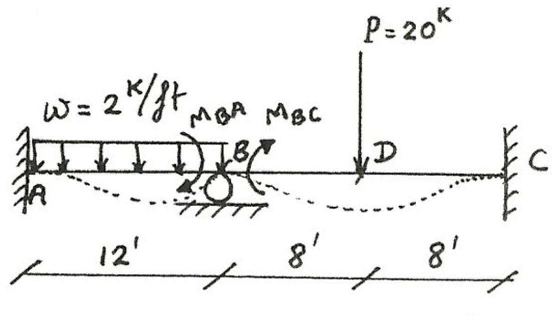
$$M_{AB} = FEM_{AB} + \left(-\frac{6EI\Delta}{L^2}\right) + \left(+\frac{4EI\theta_A}{L}\right) + \left(\frac{2EI\theta_B}{L}\right) = 2E\left(\frac{I}{L}\right)\left[2\theta_A + \theta_B - 3\left(\frac{\Delta}{L}\right)\right] + FEM_{AB}$$

$$M_{BA} = FEM_{BA} + \left(-\frac{6EI\Delta}{L^2}\right) + \left(+\frac{2EI\theta_A}{L}\right) + \left(\frac{4EI\theta_B}{L}\right)$$
$$= 2E\left(\frac{I}{L}\right)\left[2\theta_B + \theta_A - 3\left(\frac{\Delta}{L}\right)\right] + FEM_{BA}$$

Note:

$$\Psi_{AB} \equiv \text{Chord rotation of beam} = \Psi_{BA} = \left(\frac{\Delta}{L}\right) = \left({}^{+\ \text{IF CW}}_{-\ \text{IF CCW}}\right)$$

Example 1: Beam (no settlement, no-sideways effects)



Step 1: Identify all geometrical unknowns
(Such as rotation, translation at certain joints)
1 unknown = θ_B.

Step 2: Obtain the corresponding number of equation(s)
(Hint: by writing equilibrium eq(s) at the same place with the unknowns!)

$$\sum M_B = 0 \leftrightarrow M_{BA} + M_{BC} = 0 \qquad (50)$$

Step 3: Applying the slope Deflection Equation (SDE) formula to Eq.(50)

$$M_{BA} = 2E\left(\frac{I}{12}\right)\left[2\theta_B + \theta_A - 3\left(\Psi_{BA} = \frac{\Delta}{L}\right)\right] + FEM_{BA} \qquad (52)$$

Zero

$$+\frac{wL^2}{12} = \overset{+}{cw}24$$

$$M_{BC} = 2E\left(\frac{I}{16}\right)[2\theta_B + \theta_C - 3(\Psi_{BC})] + FEM_{BC} \qquad (53)$$

$$-\frac{PL}{8} = \frac{-(20^K)(16')}{8} = \overset{-}{ccw}\ 40$$

Hence: $$\underbrace{2E\left(\frac{I}{12}\right)[2\theta_B] + 24}_{M_{BA}} + \underbrace{2E\left(\frac{I}{16}\right)[2\theta_B] - 40}_{M_{BC}} = 0 \qquad (51)$$

Step 4: Solve for all unknown displacement and/or rotations (see Eq.51):

$$\theta_B = +\frac{192}{7EI} \qquad (54)$$

Step 5: Solve for all the end-moments, by referring to SDE Formula:

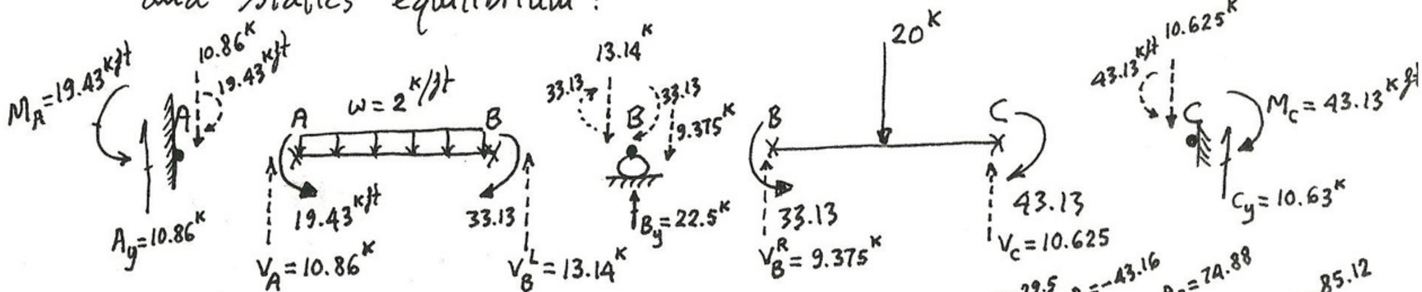
$$M_{BA} = \frac{4EI(\theta_B = \frac{192}{7EI})}{12} + 24 = 33.13^{Kft} \text{ And } M_{AB} = \frac{2EI\theta_B}{12} - 24 \overset{-}{=}_{CCW} 19.43$$

$$M_{BC} = \frac{4EI(\theta_B = \frac{192}{7EI})}{16} - 40 = -33.13 \text{ And } M_{CB} = \frac{2EI\theta_B}{16} + 40 = 43.43$$

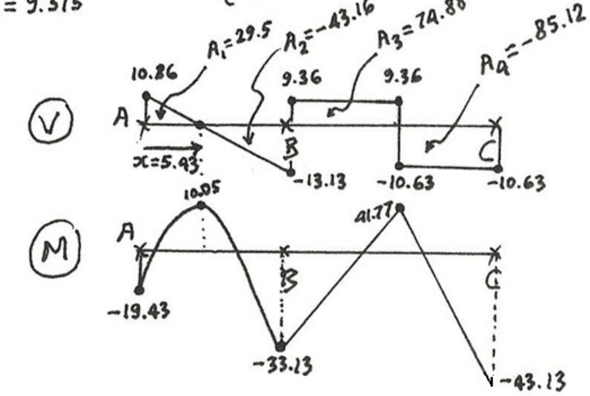
Step 6:

Support reactions can be found from Free Body Diagrams (FBD) and statics equilibrium.

and statics equilibrium :



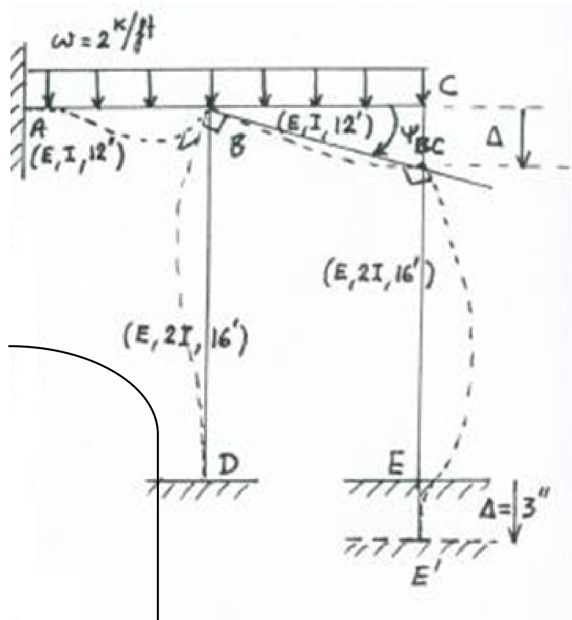
Step 7 Draw (V) & (M) Diagrams



Example 2: SDE with settlement for 2-D frame (units = K, ft.)

Settlement at support $\mathrm{E} = \ \Delta = 3'' \downarrow = \left(\frac{3}{12}\right)$ ft

$\Rightarrow$Chord rotation for member $\mathrm{BC} \ = \ \Psi_{\mathrm{BC}} = \ \frac{+3''}{12\ \mathrm{ft}} = \frac{3^{\mathrm{ft}}}{144^{\mathrm{ft}}}$



From SDE formula:

$$M_{AB} = 2E\left(\frac{I}{12}\right)[\theta_B] - \left(\frac{wL^2}{12} \equiv 24\right)$$

$$M_{BA} = 2E\left(\frac{I}{12}\right)[2\theta_B] + \left(\frac{wL^2}{12} = 24\right)$$

$$M_{BC} = 2E\left(\frac{I}{12}\right)\left[2\theta_B + \ \theta_C - \ 3\left(\Psi_{BC} = \frac{+\Delta}{L} = \frac{3}{144}\right)\right] + \left(\mathrm{FEM}_{BC} = -\frac{wL^2}{12} = -24\right)$$

$$M_{CB} = 2E\left(\frac{I}{12}\right)\left[2\theta_C + \ \theta_B - \ 3 * \left(\frac{3}{144}\right)\right] + \left(\mathrm{FEM}_{CB} = \frac{+wL^2}{12} = +24\right)$$

(55)

$$M_{BD} = 2E\left(\frac{2I}{16}\right)[2\theta_B]$$

$$M_{DB} = 2E\left(\frac{2I}{16}\right)[\theta_B]$$

$$M_{CE} = 2E\left(\frac{2I}{16}\right)[2\theta_C]$$

$$M_{EC} = 2E\left(\frac{2I}{16}\right)[\theta_C]$$

- 2 unknowns: θ_B, θ_C ?

- 2 equilibrium equations :

$$\sum M_B = 0 = M_{BA} + M_{BC} + M_{BD} \quad (56)$$

$$\sum M_C = 0 = M_{CB} + M_{CE} \quad (57)$$

- Substituting Eq. (55) into Eqs. (56, 57) , one obtains:

$$0 = f_1(\theta_B, \theta_C) \quad (58)$$

$$0 = f_2(\theta_B, \theta_C) \quad (59)$$

- Solving Eqs. (58, 59) simultaneously to obtain:

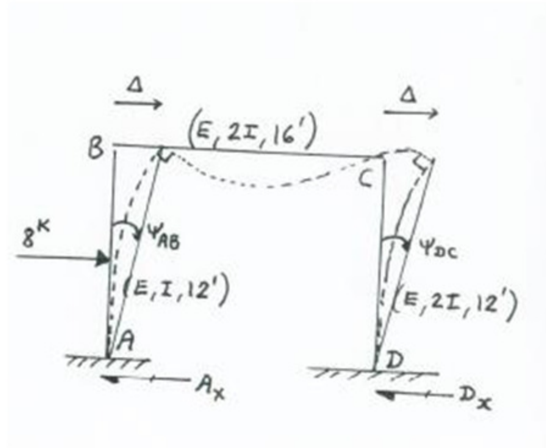
$$\theta_B = \text{known}_1 \quad (60)$$

$$\theta_C = \text{known}_2 \quad (61)$$

- Substituting Eqs. (60 & 61) into Eq. (55) to obtain all end-moments.

- From FBD of member(s) and joint(s), and using statics equilibrium Eqs., to get all support reactions!

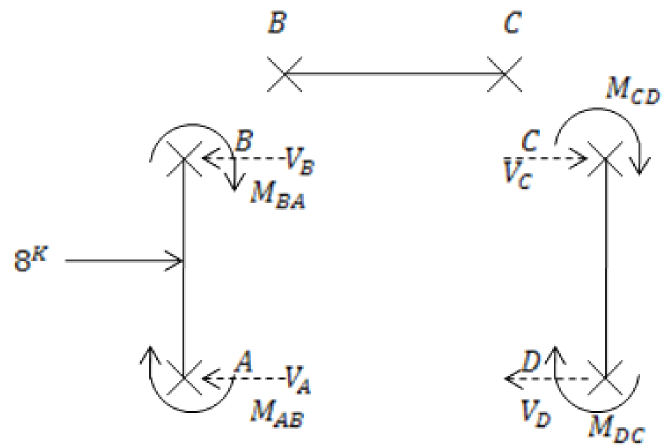
Example 3: SDE with Side-Way Effects



From SDE formula:

$$
\left.
\begin{aligned}
M_{AB} &= 2E\left(\frac{I}{12'}\right)\left[\theta_B - 3\left(\Psi_{AB} = \frac{+\Delta}{L}\right)\right] + \left(FEM_{AB} = -\frac{PL}{8} = -12\right) \\
M_{BA} &= 2E\left(\frac{I}{12}\right)\left[2\theta_B - \frac{3\Delta}{12}\right] + (12) \\
M_{BC} &= 2E\left(\frac{2I}{16}\right)[2\theta_B + \theta_C] \\
M_{CB} &= 2E\left(\frac{2I}{16}\right)[2\theta_C + \theta_B] \\
M_{DC} &= 2E\left(\frac{2I}{12}\right)\left[\theta_C - 3(\Psi_{DC} = \frac{+\Delta}{12'}\right] \\
M_{CD} &= 2E\left(\frac{2I}{12}\right)\left[2\theta_C - 3\frac{\Delta}{12}\right]
\end{aligned}
\right\} \qquad (62)
$$

Free Body Diagrams and Static Equilibrium:



- From FBD of column AB:

$$\sum M_B \overset{\circlearrowright +}{=} 0 = M_{BA} - (8^k)(6') + M_{AB} + (V_A)(12')$$

Thus: $V_A = \frac{48^{Kft} - M_{BA} - M_{AB}}{12}$ ________(63)

- From FBD of column DC:

$$\sum M_C \overset{\circlearrowright +}{=} 0 = M_{CD} + M_{DC} + (V_D)(12')$$

So: $V_D = \frac{-M_{CD} - M_{DC}}{12}$ ________________________________(64)

- Substituting Eq. (62) into Eqs. (63 and 64) to get:

$$\left.\begin{array}{l} V_A = f_1(\theta_B, \Delta) \\ V_D = f_2(\theta_C, \Delta) \end{array}\right\}$$ ________________________________(65)

- System Equilibrium at Joint B:

$$\sum M_B = M_{BC} + M_{BA} = 0 \qquad (66)$$

- System Equilibrium at Joint C:

$$\sum M_C = M_{CB} + M_{CD} = 0 \qquad (67)$$

- System $\sum F_x = +8^K - V_A - V_D = 0$

Or: $8^K - f_1(\theta_B, \Delta) - f_2(\theta_C, \Delta) = 0$ (68)

❖ Substituting Eq. (62) into Eqs. (66 and 67) to obtain:

$$2E\left(\frac{2I}{16}\right)[2\theta_B + \theta_C] + 2E\left(\frac{I}{12}\right)\left[2\theta_B - \frac{3\Delta^{ft}}{12'}\right] + 12^{Kft} = 0 \qquad (69)$$

$$2E\left(\frac{2I}{16}\right)[2\theta_C + \theta_B] + 2E\left(\frac{2I}{12}\right)\left[2\theta_C - \frac{3\Delta'}{12'}\right] = 0 \qquad (70)$$

❖ Simultaneously solve 3 Eqs. (68 →70), one obtains:

$$\begin{Bmatrix} \theta_B \\ \theta_C \\ \Delta \end{Bmatrix} = \begin{Bmatrix} -7.351/EI \\ 19.959/EI \\ 256.734/EI \end{Bmatrix} \qquad (71)$$

- Substituting Eq. (71) into Eq. (62) to get:

$M_{AB} = -23.956$ (or$_{CCW}$) And $M_{BA} = -1.214$($_{CCW}$)

$M_{BC} = +1.214^{Kft}$ (or$_{CW}$) And $M_{CB} = +8.092$

$M_{CD} = -8.092$ And $M_{DC} = -14.742$

- Draw moment M Diagram.